ASSIGNMENT SET - I

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

B.Sc Hon.(CBCS)

Mathematics: Semester-I

Paper Code: C1T

[Calculus, Geometry, Differential Equation]

Answer all the questions

1. Find the all the asymptotes of the curve x $(y - x)^2$ - x(y – x) = 2.
2. Find the horizontal asymptotes for f(x) = $\frac{(x^2+3)}{x+1}$.
3. Differentiate sin(cos x^2) with respect to x.
4. If y = sin (m$cos^{-1}\sqrt{x}$), then prove that $\lim_{x\to 0} \frac{y_{n+1}}{y_n} = \frac{4n^2-m^2}{4n+2}$.
5. Find a,b,c such that $\frac{ae^x - bcosx + ce^{-x}}{xsinx} \to 2,$ as x$\to 0$.
6. If $I_n = \int_0^1 (lnx)^n$dx, show that $I_n = (-1)^n$n!. n being positive integer.
7. Find asymptote (if any) of the curve y = a log[sec $\frac{x}{a}$].
8. Prove that cosh(x+y)=coshx cushy + sinhx sinhy.
9. Show that the curve re^{θ} = a(1+ θ) has no point of inflexion.
10. Find the point of inflexion of the curve x=a tan θ, y=a sin θ cos θ.
11. Show that the curve y=e^{-x^2} has points of inflexion at x = $\pm\frac{1}{\sqrt{2}}$.

12. If y= sin($m\sin^{-1}x$) then show that (1- x^2) y_{n+2}- (2n+1)x y_{n+1} + $(m^2 - n^2)\, y_n$ = 0.
13. If $y^{\frac{1}{m}}$ + $y^{\frac{-1}{m}}$ = 2x, then prove that ($x^2 - 1$) y_{n+2} + (2n-1)x y_{n+1} + $(n^2 - m^2)\, y_n$ = 0.
14. If $\lim_{x\to 0} \frac{a\, sinx - sin2x}{tan^3 x}$ is finite, find the value of a, and the limit.
15. Find the asymptotes of the curve x^3+ x^2y - x y^2 - y^3 + x^2 - y^2 = 2.
16. A circle moves with its centre on the parabola y^2 = 4ax and always passes through the vertex of the parabola. Show that the envelope of the circle is the curve x^3 + y^2 (x + 2a) =0 .
17. Show that the envelope of circles whose centres lie on the rectangular hyperbola xy = c^2 and which pass through its centre is $(x^2 + y^2)^2$ = 16 c^2xy.
18. The envelope of the lines $\frac{x}{a} + \frac{y}{b}$ =1 (a, b are variable parameters) is given by $\sqrt{x}$ + $\sqrt{y}$ =$\sqrt{k}$ (k is a given constant). Find the relation between a and b.
19. Find the envelope of the lines $\frac{x}{a} + \frac{y}{b}$ =1 (a, b are variable parameters), where a, b are connected by the relation a^n + b^n = c^n.
20. Prove that the asymptotes of the curve r = $\frac{a}{\frac{1}{2} - cos\theta}$ are $\sqrt{3}$ r (sinθ $\pm \sqrt{3\, cos\theta}$) $\pm$ 4a = 0.

UNIT-2

1. Obtained a reduction formula for $I_n = \int sin^n x\, dx$.
2. For any positive integer n prove that $\int_0^{\pi} \frac{sin^2 nx}{sin^2 x}$ dx = nπ.
3. Find the value of $\int sin^5 x\, dx$.
4. Obtained a reduction formula for $I_n = \int cos^n x\, dx$.
5. Obtained a reduction formula for $I_{m,n} = \int sin^m x\, sin^n x dx$.
6. Prove that $\int_0^{\frac{\pi}{2}} sin^m x\, sin^n x dx = \frac{1.3.5...(m-1).1.3.5...(n-1)}{2.4.6...(m+n)} \frac{\pi}{2}$, if both m, n are even.

7. Obtained a reduction formula for $I_{m,n} = \int cos^m x \sin nx\, dx$.(m, n positive integers)and hence evaluate $\int_0^{\frac{\pi}{2}} sin^5 x\; sin3xdx$.
8. Obtained a reduction formula for $I_{m,n} = \int cos^m x \sin nx\, dx$.(m, n positive integers), then show that $I_{m,n} = \frac{1}{m+n} + \frac{m}{m+n} I_{m-1,n-1}$.
9. If m be a positive integer prove that $\int_0^{\frac{\pi}{2}} cos^m x \sin mx\, dx = \frac{1}{2^{m+1}}$ [2 + $\frac{2^2}{2}$ + $\frac{2^3}{3}$ + ... + $\frac{2^m}{m}$].
10. Obtained a reduction formula for $\int_0^{\frac{\pi}{2}} cos^m x \cos nx\, dx$, (m, n are positive integers) , hence show that $\int_0^{\frac{\pi}{2}} cos^m x \cos mx\, dx = \frac{\pi}{2^{m+1}}$, m being positive integer.
11. Find the length of the line segment on 2y - 2x + 3 = 0 between y = 1 and y = 3.
12. Find the arc length of the curve f(x) = √x from x = 0 to x = 4.
13. Find the area of the surface generated by revolving y = $\sqrt{25-x^2}$ on the interval [-2,3] about the x-axis.
14. Find the area of the surface generated by revolving x = 1 - t^2, y = 2t, on the t-interval [0,1] about the x-axis.
15. Find the length of the curve shown in Figure below, which is the graph of the function $y = \frac{4\sqrt{2}}{3} x^{\frac{3}{2}} - 1, \quad 0 \leq x \leq 1$.
16.: Find the length of the curve $y = (x/2)^{2/3}$ from x = 0 to x = 2 .
17. Using the definition, find the length of the circle of radius r (circumference) defined parametrically by: $x = r$ cos t and $y = r$ sin t, $0 \leq t \leq 2\pi$.
18. Find the length of the asteroid given in the Figure below, and defined parametrically by the following equations: x = cos3 t and y = sin3 t, $0 \leq t \leq 2\pi$.
19. Find the length of the curve x = 8cos t + 8t sin t and y = 8 sin t − 8t cos t, $0 \leq t \leq \pi/2$
20. Find the length of the curve $x = t^3$and $y = \frac{2t^2}{2}$, $0 \leq t \leq \sqrt{3}$.

UNIT-3

1. Find the equation of the right circular cylinder whose axis is $\frac{x}{1} = \frac{y}{-2} = \frac{z}{2}$ and radius is 2.
2. Find the polar equation of the normal to the conic $\frac{l}{r} = 1 + e\cos\theta$.
3. Find the equation of the generator of the cone $x^2 + y^2 = z^2$ through the point (3, 4, 5).
4. Find the equations of the straight lines in which the plane 2x + y – z = 0 cuts the cone $4x^2 - y^2 + 3z^2 = 0$.
5. On the ellipse r (5 - 2cosθ) 21 , find the point with the greatest radius vector.
6. Reduction the equation $x^2 + 4xy + y^2 - 2x + 2y + 6 = 0$, to its canonical form and show that it represents a hyperbola. Find the latus rectum and equation of the exist of the hyperbola.
7. Reduction the equation $3x^2 + 10xy + 3y^2 - 12x - 12y + 4 = 0$ to its canonical form. and determine the type of the conic represented by it.
8. Show that the auxiliary circle of the conic $\frac{l}{r} = 1 - e\cos\theta$ is $r^2(e^2 - 1) + 2ler\cos\theta$.
9. Show that the locus of the poles of the normal cords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{a^6}{x^2} + \frac{b^6}{y^2} = (a^2 - b^2)^2$
10. Find the equation of the conjugate diameter of the hyperbola $4x^2 - 5y^2 = 20$, if one of them passes through the point (1,8).
11. Prove that the equation of the sphere circumscribing the tetrahedron whose faces are $\frac{y}{b} + \frac{z}{c} = 0$, $\frac{z}{c} + \frac{x}{a} = 0$, $\frac{x}{a} + \frac{y}{b} = 0$, and , $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ is $\frac{x^2+y^2+z^2}{a^2+b^2+c^2} - \frac{x}{a} - \frac{y}{b} - \frac{z}{c} = 0$.
12. Reduce the equation $11x^2 + 4xy + 14y^2 - 26x - 32y + 23 = 0$ to standard form and hence write the eccentricity of the conic represented by it.

13. Reduce the equation 4x^2 + 4xy + y^2 -12x -6y +5 =0 to its canonical form and determine its nature.
14. Find the locus of the centre of the sphere touching both the straight lines y=mx, z= c and y= -mx, z= -c.
15. Find the points on the sphere $x^2 + y^2 + z^2$ - 2x -2y -4z +2=0 which is nearest and for these from the point (2, -1, 3). Also find the least and greatest distance of (2, -1, 3) from the sphere.
16. If a plane passes through a fixed point (α, β, γ) and cuts the axes at P, Q, R, show that the locus of the centre of the sphere passing through the origin and the point P, Q, R is $\frac{\alpha}{x} + \frac{\beta}{y} + \frac{\gamma}{z} = 2$.
17. A variable sphere passes through the points (0, 0 , $\pm c$) and cuts the straight lines y = x tan α, z= c and y = -x tan α, z= -c at the point P and P' other than (0, 0 , $\pm c$). If PP'= 2a(constant), show that the centre of the sphere lies on the circle $x^2 + y^2 = (a^2 - c^2)\, cosec^2 2\alpha$, z = 0.
18. Reduce the equation 4x^2 + 4xy + y^2 - 4x -2y + a = 0 to its canonical form and determine its nature for different values of a.
19. Show that the equation x^2 + 2xy + y^2- 4x +8y -6 = 0 represent a parabola whose directrix is 3x – 3y +8 = 0.
20. A sphere of radius r passes through the origin and meets the axes in P, Q, R. Find the locus of the centroid of the triangle PQR.
21. A sphere touches the planes 2x + 3y – 6x + 14 = 0, and 2x + 3y – 6x + 42 = 0 and its centre lies on the line 2x + z 0 , y=0. Find the equation of the sphere.
22. A sphere S has points (0, 1, 0) and (3, -5, 2) as a possible ends of a diameter. Find the equation of the sphere on which the intersection of the plane 5x – 2y + 4z + 7= 0, with the sphere S is a great circle.
23. A sphere of constant radius r passes through passes through the origin and cuts he axes at A, B, C. Prove that the locus of the foot of perpendicular from O on the plane ABC is given by $(x^2 + y^2 + z^2)^2\, (x^{-2} + y^{-2} + z^{-2})$ = 4r^2.

UNIT-4

1. Determine the order and the degree of the differential equation
   i. $\frac{d^3y}{dx^3} + (\frac{d^2y}{dx^2})^5 + (\frac{dy}{dx})^4 + 5y = 8x$.
2. Find the differential equation of the following y = a cosx + b sinx; a, b are arbitrary constants.
3. State existence and uniqueness theorem of the solution of a first order and first degree differential equation.

4. Examine the differential equation is exact or not $(e^x \sin y + e^{-y})dx + (e^x \cos y - x\, e^{-y})dy = 0$.

5. Solve: $(x+y)^2\, dx - (y^2 - 2xy - x^2)\, dy = 0$.

6. Define integrating factor.
7. Find integrating factor of the differential equation $(x^2 + y^2)dx - 2xy\, dy = 0$.
8. Solve: $\frac{2x}{y^3} dx + \frac{y^2 - 3x^2}{y^4} dy = 0$.
9. Solve : $(2x + \tan y)\, dx + (x - x^2 \tan y)\, dy = 0$.
10. Solve : $(2xy^2 + y)\, dx + (2y^3 - x)\, dy = 0$.
11. Solve: $(y^4+2y)dx + (xy^3+2y^4-4x)dy = 0$.
12. Find the differential equation of the following
13. y = a cos(logx) + b sin(logx); a, b are arbitrary constants.
14. Show that the differential equation of all parabolas with foci at the origin and axes along the $x-$ axis is $y\,(\frac{dy}{dx})^2 + 2x\frac{dy}{dx} - y = 0$.
15. Prove that the differential equation of the circles through the intersection of the circle $x^2 + y^2 = 1$ and the line x – y =0 is $(x^2 - 2xy - y^2 + 1)\, dx + (x^2 + 2xy - y^2 - 1)\, dy = 0$.
16. If y_1 and y_2 solution of the equation $\frac{dy}{dx} + P(x)\, y = Q(x)$, and $y_2 = z\, y_1$, than show that $z = 1 + a\, e^{-\int\left(\frac{Q}{y_1}\right)dx}$, where a is arbitrary constant.

17. Show that the general solution of the differential equation $\frac{dy}{dx}$ + P(x) y = Q(x), can be written in the form y = $\frac{Q}{P}$ - $e^{-\int P\,dx}$[$e^{\int P\,dx}$d($\frac{Q}{P}$) + c], where c is an arbitrary constant.
18. If $\frac{1}{M-N}$ ($\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}$) = f(x + y), than differential equation M dx + N dy =0 has an integrating factor of the form $e^{-\int f(x+y)d(x+y)}$.
19. Show that the general solution of the differential equation $\frac{dy}{dx}$ + P(x) y = Q(x), can be written in the form y = k(f - g) + g, where k is a arbitrary constant and f,g be its particular solutions.
20. Reduce the differential equation y^2(y - px) = x^4 p^2 to Clairaut's form by the substitutions u = $\frac{1}{x}$, v= $\frac{1}{y}$, solve it for singular solution and extraneous loci, if any.
21. Use the transformation y = x^2 and v = y^2 to solve the equation (px - y)(py + x) = h^2p.

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